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A lattice QCD calculation of the transverse decay constant of the $b_1(1235)$ meson

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ABSTRACT

We review various B meson decays that require knowledge of the transverse decay constant of the $b_1(1235)$ meson. We report on an exploratory lattice QCD calculation of the transverse decay constant of the b_1 meson. The lattice QCD calculations used unquenched gauge configurations, at two lattice spacings, generated with two flavours of sea quarks. The twisted mass formalism is used.

1. Introduction and motivation

The transverse decay constant of the $b_1(1235)$ meson ($f^T_{b_1}$) is theoretical input to a number of decays of the B meson. For example $f^T_{b_1}$ is an important QCD input to the following decays: $\overline{B}^0 \to b_1^- \rho^+$, $\overline{B}^0 \to b_1^- K^{\star +}$, using the light cone formalism [1,2]. The $f^T_{b_1}$ constant is also input to the decay $B \to b_1 \gamma$ [3,4] that use light cone sum rules. Diehl and Hiller [5] discuss studying decays of the B meson with final states that include the b_1 meson.

There are alternative theoretical formalisms [6–9] to the light cone sum rules such as factorisation, that describe the non-leptonic decays of the B meson to final states that include the b_1 meson. It is important to compare the different formalisms and this requires accurate input parameters, such as $f^T_{b_1}$ for light cone sum rules.

BaBar has experimentally measured the B decays: $b_1\pi$ and $b_1 K$ [10,11]. The charmless decays of the B meson, that include those with a b_1 meson in the final state, have been reviewed by Cheng and Smith [12].

The transverse decay constant of the b_1 meson is not accessible to experiment, but can be calculated in models [13] and sum rules. Calculations of the b_1 meson have also been used to tune sum rules [14–16]. In particular, the same sum rules are used to simultaneously extract the transverse decay constants of the b_1 and ρ mesons [14,15].

In principle lattice QCD should be able to produce an accurate result for $f^T_{b_1}$, particularly as modern lattice QCD calculations usually have multiple lattice spacings and volumes, with pion masses below 300 MeV [17,18]. To the best of our knowledge, there has never been a lattice QCD calculation of $f^T_{b_1}$ before this one. In this Letter we report an exploratory lattice QCD calculation of $f^T_{b_1}$.

The b_1 meson is a good test case for lattice techniques that deal with resonances, because it is thought to be a basic quark–antiquark meson that decays via S-wave. The experimental width of the $b_1(1235)$ is 142(9) MeV, and the bulk of the decays are to $\omega\pi$. Hence further motivation for this study is to compute as much information about the b_1 meson from our lattice QCD calculations as possible.

Light cone sum rules and factorisation methods, also use the decay constants of the a_0, $\pi(1300)$, a_1 mesons to study the decays of the B meson, but there have been previous lattice QCD calculations of those quantities [19–21].

2. The lattice QCD calculation

The transverse decay constant ($f^T_{b_1}(\mu)$) of the b_1 meson is defined [22] by

$$\langle 0 | \overline{\psi} \sigma_{\mu\nu} \psi | b_1(P, \lambda) \rangle = i f^T_{b_1}(\mu) \epsilon_{\mu\nu\alpha\beta} \epsilon^{\alpha}_{(\lambda)} P^{\beta} \tag{1}$$

where $\sigma_{\mu\nu} = i/2[\gamma_\mu, \gamma_\nu]$, and $\epsilon^{\alpha}_{(\lambda)}$ is the polarisation vector of the meson. It is convenient to introduce the tensor current $T_{\nu\mu} = \overline{\psi}\sigma_{\mu\nu}\psi$. We do not include any momentum in this lattice calculation. For completeness we note that the b_1 meson has $J^{PC} = 1^{+-}$.

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Table 1
Summary of results for $f_{b_1}^T$ used in this calculation. $a\mu_q$ is the bare mass of the light quark in lattice units. The ensemble names are from [31].

Ensemble	β	$a\mu_q$	Volume	$f_{b_1}^T$ (2 GeV)
B_1	3.9	0.004	$24^3 \times 48$	249(55)
B_2	3.9	0.0064	$24^3 \times 48$	239(29)
B_3	3.9	0.0085	$24^3 \times 48$	254(26)
B_4	3.9	0.0100	$24^3 \times 48$	220(32)
B_5	3.9	0.0150	$24^3 \times 48$	256(33)
B_6	3.9	0.004	$32^3 \times 64$	233(29)
C_1	4.05	0.003	$32^3 \times 64$	202(83)
C_2	4.05	0.006	$32^3 \times 64$	193(55)
C_3	4.05	0.008	$32^3 \times 64$	216(44)
C_4	4.05	0.012	$32^3 \times 64$	289(47)

In the isospin limit the leptonic decay constant of the b_1 meson is zero, because the T_{ij} operator is orthogonal to the vector and axial currents. Although for some quantities, such as the matrix element for ρ–ω mixing [23] or the decay constant of the flavour non-singlet 0^{++} meson [20] an estimate of isospin violating quantities can be made from a lattice calculation with two degenerate flavours of sea quarks, we don't see how to estimate the leptonic decay constant of the b_1 meson without using non-degenerate light quarks. This could be done with the twisted mass-split formalism of Frezzotti and Rossi [24,25].

Our lattice calculation uses the twisted mass QCD formalism with degenerate light quarks [26]. Once a single parameter has been tuned, twisted mass QCD has non-perturbative $O(a)$ improvement [27,28] for physical quantities. Twisted mass QCD has been found to have small $O(a^2)$ errors for properties of hadrons [29] made from light quarks, with the exception of the mass of the neutral pion (see [28] for a theoretical discussion of this). The twisted mass formalism has been reviewed by Shindler [30].

We have recently reported on the some basic measurements of the ρ, b_1 and a_0 mesons [32]. In this Letter we extend that study to the transverse decay constant of the b_1 meson. All the necessary details are in the previous paper [32] and here we provide a brief summary. There is a additional information about the lattice techniques, such as the smearing and variational analysis in the "methods paper for the ETM Collaboration" [33]. We used the twisted mass Wilson action and the tree level improved Symanzik action. The ensembles used in this analysis are in Table 1. Details of the analysis of light pseudo-scalar mesons are in [33,34].

The correlators used to extract the $f_{b_1}^T$ decay constant are in Eq. (2). We use a smearing matrix of order 2, using basis functions of local and fuzzed interpolating operators, that includes the correlators in Eq. (2). We fit the smearing matrix to a factorising fit form [33] with two states.

$$\sum_x \sum_{k=1,\, i<j}^{3} \epsilon_{ijk} \langle T_{ij}(x,t_x) T_{ij}(0,0)^\dagger \rangle \to \frac{3 m_{b_1} (f_{b_1}^T)^2 e^{-m_{b_1} t_x}}{2} \qquad (2)$$

It is particularly important to use fits with two exponentials for the correlators of the b_1 meson. The correlators for the b_1 meson are more noisy than for lighter states, such as the ρ meson (see the recent results from the Hadron Spectrum Collaboration for example [35]), so the fit needs to start early before the signal is lost in the noise. If the tuning to maximal twist is not exact, the b_1 correlator ($J^{PC} = 1^{+-}$) could couple to the correlator of the ρ meson with the opposite parity. We found that the lightest mass in the b_1 channel was much higher than the mass of the vector meson [32]. The charged interpolating operator for the b_1 meson in the twisted basis was used [33].

We used ensembles at two different β values. At $\beta = 3.9$ we included two volumes. We used the pion decay constant of the

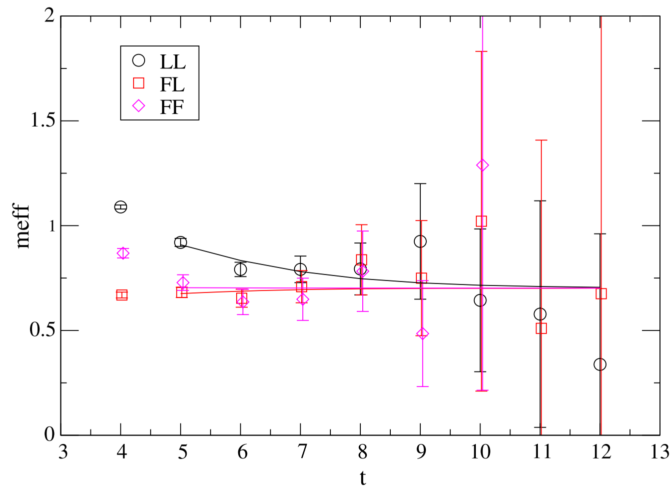

Fig. 1. Effective mass plot for the b_1 channel at $\beta = 3.9$, $\mu_q = 0.004$, $L = 24$. The labels LL, FL, FF refer to local–local, fuzzed–local, and fuzzed–fuzzed.

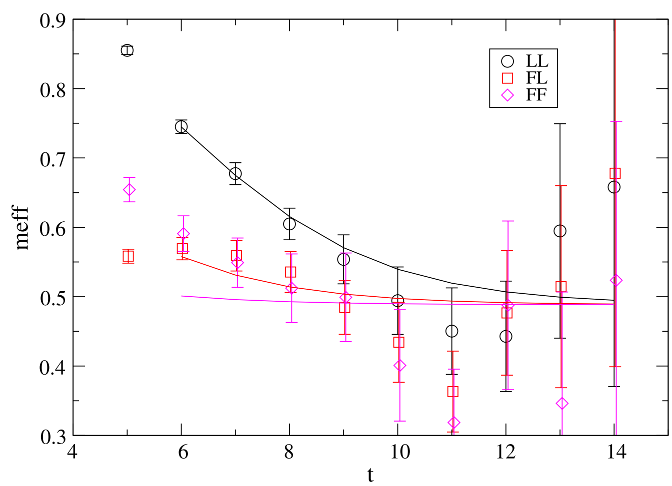

Fig. 2. Effective mass plot for the b_1 channel at $\beta = 4.05$, $\mu_q = 0.006$, $L = 32$. The labels are the same as for Fig. 1.

π meson to determine the lattice spacing. At $\beta = 4.05$ ($\beta = 3.9$) the lattice spacing is: $a = 0.0667(5)$ fm ($a = 0.0855(5)$ fm). The lattice spacing from f_π was consistent with the value from the nucleon mass [36]. In Figs. 1 and 2 we show effective mass plots for the b_1 correlators. The continuous lines in Figs. 1 and 2 show the fitted correlator. As we discussed in Section 2, the correlators for the b_1 meson are more noisy than the correlators for the ρ meson (compare Figs. 1 and 2 to the effective mass plot in [32]). All the χ^2/dof for the fits were less than 1, with the correlation matrix regulated with the method described in [37]. To compute the transverse decay constant of the ρ meson, a ratio of correlators can be used that reduces the statistical errors [38]. Unfortunately no equivalent ratio exists for the transverse decay constant of the b_1 meson, so it must be directly extracted from fitting equation (2).

The decay constant $f_{b_1}^T$ depends on the value of the renormalisation scale. We used a renormalisation factor obtained from the Rome–Southampton non-perturbative method [39–41]. As traditional in lattice QCD calculations we quote the result at the scale of 2 GeV. The renormalisation group equations can be used to evolve the decay constant to another scale.

The results for the decay constant from this calculation are in Table 1. In Fig. 3 the transverse decay constant of the b_1 meson is plotted in physical units as a function of the square of the pion

Table 2
Summary of calculations of $f^T_{b_1}$ at 1 GeV.

Group	Method	$f^T_{b_1}$ (1 GeV) MeV
Chizhov [13]	extended NJL quark model	175(9)
Ball and Braun [47]	sum rule	180(20)
Bakulev and Mikhailov [48,49]	sum rule	184(5)
Bakulev and Mikhailov [48,49]	sum rule	181(5)
Yang [50,22]	sum rule	180(8)
This calculation	lattice QCD	198(36)(60)

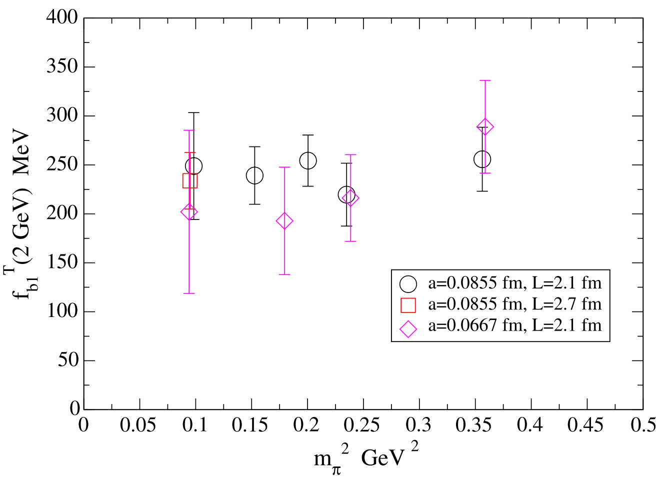

Fig. 3. The decay constant of the b_1 meson as a function of the square of the pion mass.

mass. Although the error bars are large, the results for $f^T_{b_1}$ are consistent between the two lattice spacings and volumes.

A common technique to check whether a state is a scattering state or a resonance is look at the volume dependence of the amplitude [42]. The use of the volume dependence of the amplitude and the connection to Lüscher's method has recently been discussed by Meng and Liu [43]. The "rule of thumb" is that if our b_1 correlator couples to a scattering state of $\omega\pi$, the volume dependence of the $f^T_{b_1}$ decay constant extracted from Eq. (2) is

$$f^T_{b_1} \sim \frac{1}{\sqrt{V}} \tag{3}$$

where V is the spatial volume. For a resonance $f^T_{b_1}$ should be independent of the volume (apart from small corrections if the box size is too small to fit the resonance state).

The numerical results at $\mu_q = 0.004$ at $\beta = 3.9$ (ensembles B_1 and B_6), show that $f^T_{b_1}(L=32)/f^T_{b_1}(L=24) = 0.94(24)$, compared to the prediction for scattering states in Eq. (3) of 0.65. The ratio of amplitudes is only 1.5σ from the prediction for scattering states.

In our previous paper [32] we showed that the decay of the b_1 meson to $\omega\pi$ was open in our calculation. However the mass of the lightest state in the 1^{-+} channel didn't track the sum of the masses of the ω and π mesons particularly well. In Lüscher's formalism for the study of resonances on the lattice the opening of strong decays is described by an avoided level crossing [44]. The detailed calculations of Bernard et al. [45] for the Δ baryon suggested that the avoided level crossing is "washed out" by the dynamics, so comparing the mass from the resonant interpolating operator to the sum of the masses of the decay products is probably too simplistic. A similar situation happened with string breaking, where the linearly rising potential was seen to increase beyond the energy that allowed the string to break to two static B mesons [46].

The above considerations suggest that although we don't have full control over the resonant nature of the b_1 meson, our interpolating operators are coupling to the b_1 meson in the range of quark masses in our calculations. We extrapolate $f^T_{b_1}$ linearly in the square of the pion mass to get $f^T_{b_1}(2\text{ GeV}) = 236(23)$ MeV at the physical pion mass at $\beta = 3.9$. Using the same procedure at $\beta = 4.05$ for ensembles C_1, C_2, and C_3, we obtain $f^T_{b_1}(2\text{ GeV}) = 181(33)$ MeV. With just two points we do not attempt a continuum extrapolation, so we quote the result at $\beta = 4.05$ as our central value, and quote the difference between the results at $\beta = 4.05$ and $\beta = 3.9$ as an additional systematic error due to lattice spacing errors. So our final result is $f^T_{b_1}(2\text{ GeV}) = 181(33)(55)$ MeV.

In Table 2 we collect together other estimates for $f^T_{b_1}$. We have evolved our lattice result to the scale of 1 GeV to compare with the results from sum rules. The formalism to evolve the decay constant with scale is described [32], that uses input from perturbative calculations by Gracey and others [51–54]. The perturbative factor is 1.095 to evolve $f^T_{b_1}$ from 2 GeV to 1 GeV.

In the non-relativistic quark model the b_1 meson is a P-wave meson with a node in the wave-function at the origin. This would suggest the transverse decay constant is small. The partial inclusion of relativistic effects in the quark model [55] increases the decay constant. The results in Table 2 show that $f^T_{b_1}$ is of the same order of magnitude as the pion decay constant (132 MeV), so this is evidence that local interpolating operators will couple to the b_1 meson. Pragmatically using derivative sources [56,57] with smearing techniques, such as Jacobi, may be useful to get a good signal.

3. Conclusion

We have presented the first calculation of the transverse decay constant of the b_1 meson from lattice QCD. We obtain $f^T_{b_1}(2\text{ GeV}) = 181(33)(55)$ MeV at the physical pion mass, where the first error is due to statistics and the second error is the error from the lattice spacing. Future lattice QCD calculations need to reduce the statistical errors on the correlators, take the continuum limit, and to directly take into account the resonant nature of the b_1 meson. In particular, the statistical errors need to be reduced before the lattice results for $f^T_{b_1}(2\text{ GeV})$ can replace the number from sum rules in applications. Some ideas on how to achieve this are in the study [58] by the NPLQCD Collaboration.

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